SYDNEY BOYS HIGH SCHOOL

NESA Number:

Name:

Maths Class:

2022

YEAR 12
TASK 4
TRIAL HSC

Mathematics Advanced

General Instructions

- Reading time – 10 minutes
- Working time – 3 hours
- Write using black pen
- NESA approved calculators may be used
- A reference sheet is provided with this paper
- Marks may **NOT** be awarded for messy or badly arranged work
- For questions in Section II, show ALL relevant mathematical reasoning and/or calculations

Total Marks: 100

Section I – 10 marks (pages 2 – 5)

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II – 90 marks (pages 6 – 35)

- Attempt all Questions in Section II
- Allow about 2 hours and 45 minutes for this section

Examiner: AW

Section I

10 marks
Attempt Questions 1–10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1–10

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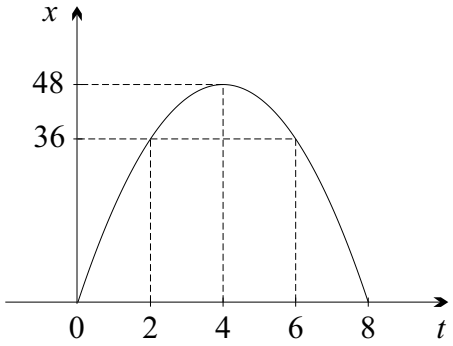
1 The time taken for an orchard to be harvested varies inversely to the number of people picking fruit.
If the orchard can be harvested by 35 people in 6 days, how many people would be needed to harvest the orchard in 5 days?

A. 42 B. 45 C. 48 D. 51

2 A ball is thrown vertically upwards from ground level and lands 8 seconds later in the same place.
Its motion is described by the equation $x = 3t(8-t)$, with displacement x metres and time t seconds. The displacement-time graph is sketched below.



What is the distance travelled by the ball in the first six seconds of motion?
A. 12 m B. 36 m C. 60 m D. 84 m

3 Which of the following is a primitive of xe^{x^2}?

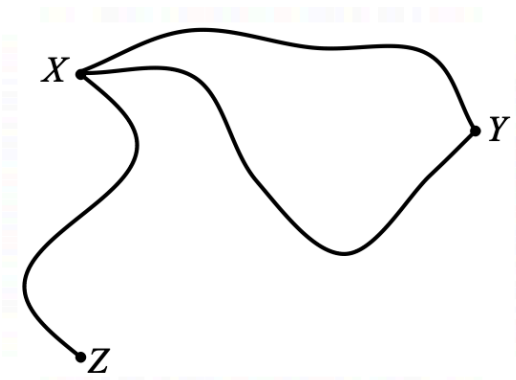
A. e^{x^2} B. xe^{x^2} C. $2e^{x^2}$ D. $\frac{1}{2}e^{x^2}$

4 If $f(x) = 2 + 5\cos\left(3x + \frac{\pi}{6}\right)$, what is the range of f?

A. $-5 \le y \le 5$ B. $-3 \le y \le 7$

C. $3 \le y \le 7$ D. $-1 \le y \le 5$

5 The figure shows three towns *X*, *Y* and *Z*.
There are three roads out of town *X* and two roads out of town *Y*.



Mr Kent and Mr Wayne leave town *X* and town *Y* respectively at the same time.
If they select their ways randomly, what is the probability that they will meet on their way?

A. $\frac{1}{6}$ B. $\frac{1}{3}$ C. $\frac{1}{2}$ D. $\frac{2}{3}$

6 A certain function $f(x)$ has the following properties: $f(0)=-1$ and $\lim\limits_{x\to\infty} f(x)=-3$.

Which of the following is possible for all values of x?

A. $f''(x)<0$ and $f'(x)>0$
B. $f''(x)<0$ and $f'(x)<0$
C. $f''(x)>0$ and $f'(x)>0$
D. $f''(x)>0$ and $f'(x)<0$

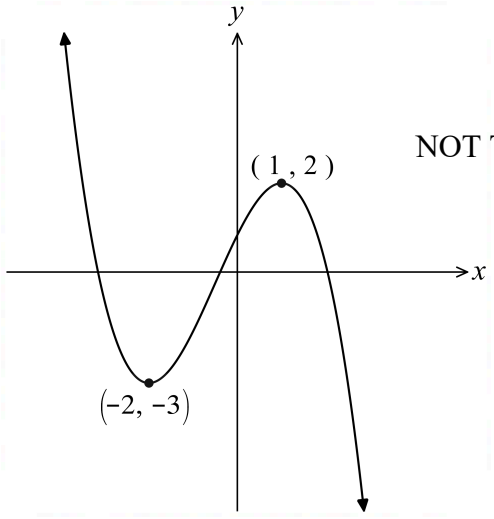
7 A sequence is defined by the recurrence relation

$$u_{n+1}=0.3u_n+6 \text{ with } u_{10}=10.$$

What is the value of u_{12}?

A. 6.6 B. 7.8 C. 8.7 D. 9.6

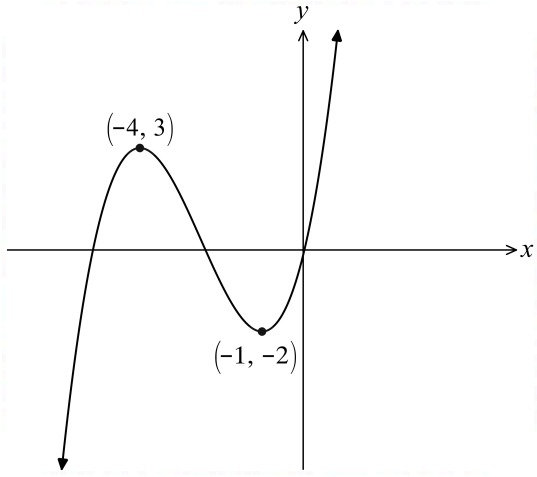
8 The diagram shows the graph of $y = f(x)$.



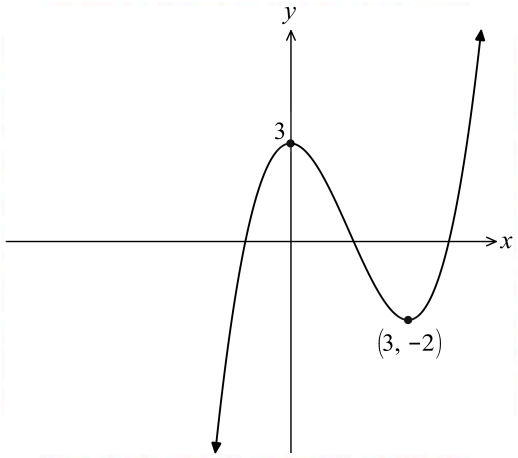
NOT TO SCALE

Which of the following shows the graph of $y = f(x+2) - 1$?

A.



B.



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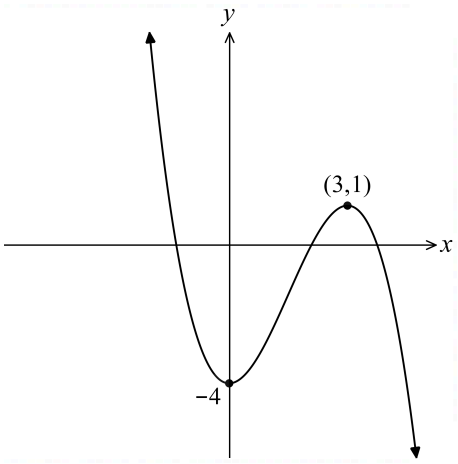
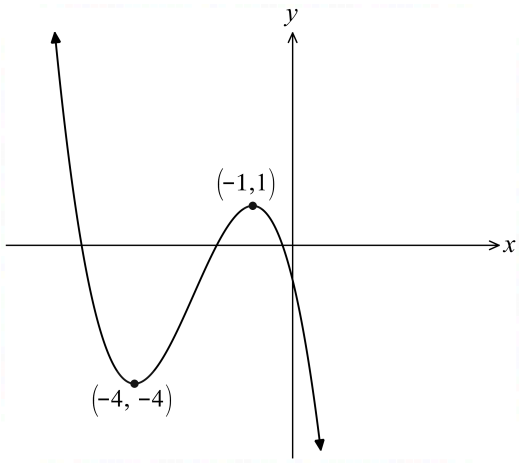


D.



9 Which of the following is $\int \frac{2x}{2x-1}\,dx$?

A. $x+\frac{1}{2}\ln\left|2x-1\right|+C$

B. $x+\ln\left|2x-1\right|+C$

C. $\ln\left|2x-1\right|+C$

D. $\frac{1}{2}\ln\left|2x-1\right|+C$

10 By considering symmetry, which of the following is equivalent to $\int_{-\frac{\pi}{2}}^{\frac{3\pi}{2}} x^3 \sin x\,dx$?

A. 0

B. $2\int_{0}^{\pi} x^3 \sin x\,dx$

C. $2\int_{0}^{\frac{\pi}{2}} x^3 \sin x\,dx+\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} x^3 \sin x\,dx$

D. $\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} x^3 \sin x\,dx$

Section II

Part A 17 marks
Attempt Questions 11–18

Answer each question in the space provided. A blank page is provided at the end of this question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 11 (1 mark)

Solve $2^x = \frac{1}{16}$. **1**

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Question 12 (1 mark)

Factorise $x^3 + 27$. **1**

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Question 13 (2 marks)

Simplify $\frac{x}{2} - \frac{x-1}{3}$. **2**

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Question 14 (1 mark)

Find a primitive of $\sqrt{x}$. **1**

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Question 15 (2 marks)

Solve $\cos\theta = -\frac{1}{2}$ for $0 \le \theta \le 2\pi$. **2**

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Question 16 (3 marks)

Calculate the sum of the arithmetic series $32 + 36 + 40 + ... + 144$. **3**

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Question 17 (4 marks)

Differentiate the following functions with respect to x.

(a) $x \tan 2x$ **2**

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(b) $\dfrac{\ln x}{x^2}$ **2**

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Question 18 (3 marks)

The parabola $y = ax^2 + bx + c$ has a vertex at (3, 1) and passes through (0, 0).

(a) Find the other x-intercept. **1**

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(b) Find a, b and c **2**

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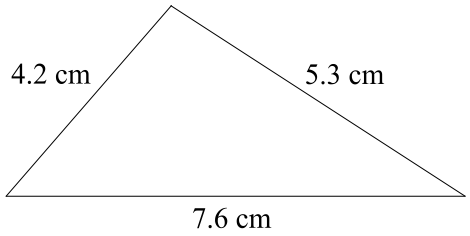
Section II

Part B 15 marks
Attempt Questions 19–23

Answer each question in the space provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 19 (3 marks)



The lengths of the sides of a triangle are 4.2 cm, 5.3 cm, and 7.6 cm.

(a) Calculate the size of the largest angle of the triangle. **2**
Leave your answer correct to the nearest degree.

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(b) Hence, calculate the area of the triangle. **1**
Leave your answer correct to one decimal place.

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Question 20 (5 marks)

Two events A and B are such that $\mathrm{P}(A) = p$, $\mathrm{P}(B) = 2p$, $\mathrm{P}(A \cup B) = 0.42$, and $\mathrm{P}(A \cap B) = 0.03$.

(a) Show that $p = 0.15$ **1**

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(b) By use of a Venn diagram, or otherwise, show that $P(A \cup \tilde{B}) = 0.73$, where $\tilde{B}$ is the complement of B. **2**

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(c) Show that $P(A \cap \tilde{B}) = 0.12$ and hence determine, with reasoning, whether the events A and $\tilde{B}$ are independent. **2**

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Question 21 (2 marks)

Evaluate $\int_{-1}^{1} x^2 + x \, dx$. **2**

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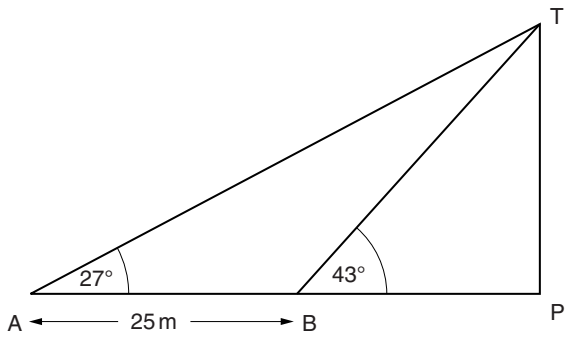
Question 22 (2 marks)

TP is a tower on horizontal ground, *ABP*. Note $\angle TPA = 90°$ **2**
Two straight cables, *AT* and *BT*, connect the top, *T*, of the tower to the ground.



AT and *BT* make angles with the ground of 27° and 43° respectively.
BT is 25 m closer to the foot of the tower, *P*, than *A*.
What is the length of the cable *AT*, correct to 2 significant figures?

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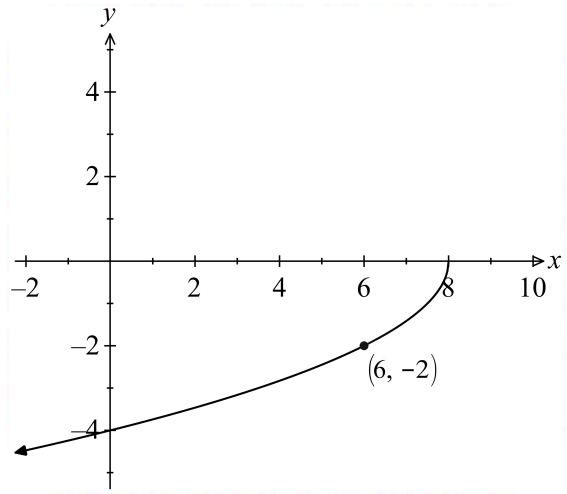
Question 23 (3 marks)

The function $f(x) = \sqrt{x}$ is transformed and the equation of the new function is of the form **3**
$y = kf(ax + b)$, where *k*, *a*, and *b* are integer constants. The graph of the new function is shown below.



What are the values of *k*, *a*, and *b*?

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Section II

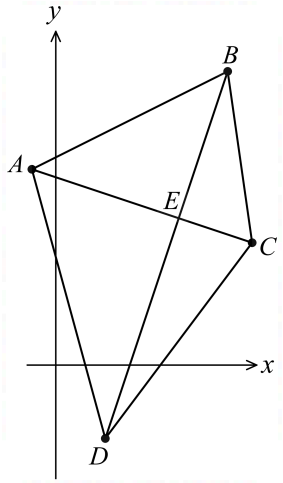
Part C 14 marks
Attempt Questions 24–27

Answer each question in the space provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 24 (7 marks)

A quadrilateral has vertices $A(-1,8)$, $B(7,12)$, $C(8,5)$, and $D(2,-3)$ as shown below.



NOT TO SCALE

(a) Show that the equation of diagonal BD is $y = 3x - 9$. **2**

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(b) The equation of diagonal AC is $x + 3y = 23$. **2**
Find the coordinates of E, the point of intersection of the diagonals.

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Question 20 continues on page 18

Question 24 (continued)

(c) Show that E lies on the perpendicular bisector of AB. **3**

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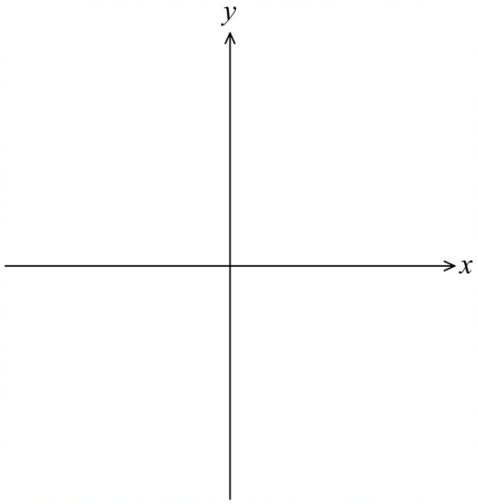
Question 25 (2 marks)

By drawing two appropriate graphs determine how many roots there are of the equation $e^x + 2x - 3 = 0$. **2**



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Question 26 (2 marks)

By using the Reference sheet, or otherwise, find $\int \frac{x^2}{\left(8-x^3\right)^{\frac{1}{2}}}\,dx$. **2**

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Question 27 (3 marks)

The table below shows the future value of a $1 annuity.

Future value of $1						
End of Year	1%	2%	3%	4%	5%	6%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746

(a) What would be the future value of a $24 000 per year annuity at 5% per annum, for two years, with interest compounding annually? **1**

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(b) An annuity of $31 800 is invested every half-year at 4% per annum, compounded six-monthly for two years. What is the future value of the annuity? **1**

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(c) Victoria aims to have a deposit for an apartment of at least $92 000 in four years time by investing in an annuity. The annuity has an interest rate of 6% per annum compounded annually. **1**
Calculate Victoria's yearly contribution, to the nearest dollar, to achieve the deposit.

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Section II

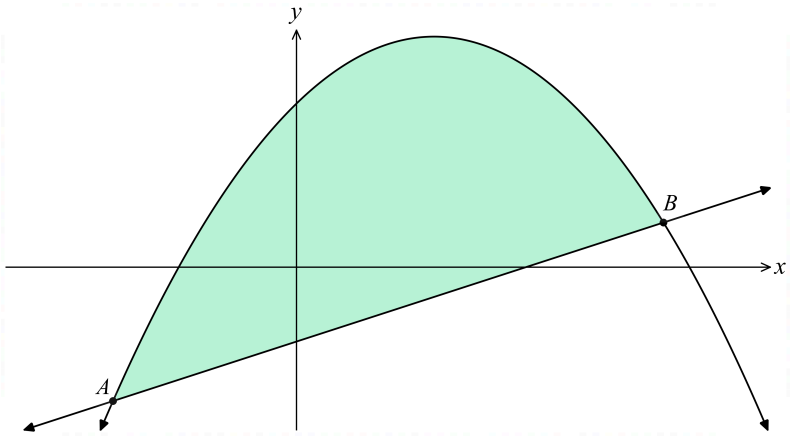
Part D **14 marks**
Attempt Questions 28–30

Answer each question in the space provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 28 (4 marks)

The diagram below shows the intersection of the graphs $y = -2x^2 + 6x + 11$ and $y = 2x - 5$ at the points $A\left(-2, -9\right)$ and $B\left(4, 3\right)$. **3**



Find the area of the shaded region.

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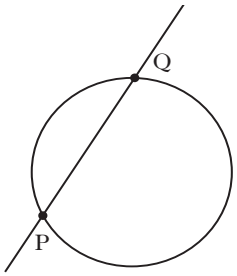
Question 29 (5 marks)

(a) Relative to a suitable set of coordinate axes, the diagram shows the line $2x - y + 5 = 0$ intersecting the circle $x^2 + y^2 - 6x - 2y - 30 = 0$ at the points P and Q. **3**

If the coordinates of P are $(-3, -1)$, find the coordinates of Q.



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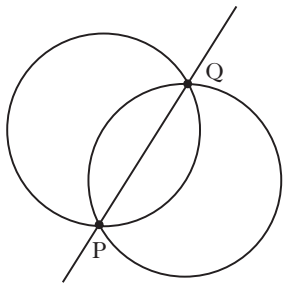
(b) The diagram below shows the circle from (a) and a second congruent circle, which also passes through P and Q. **3**

By considering the properties of rhombi, or otherwise, find the equation of the second circle.



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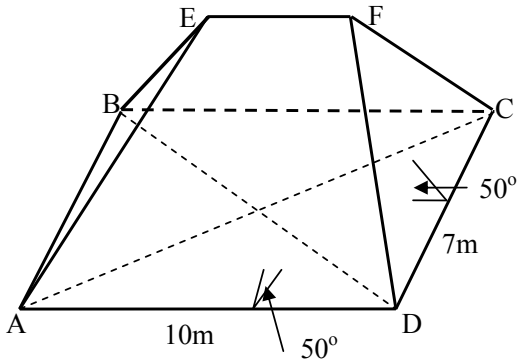
Question 30 (5 marks)

A roof of a house has a rectangular base 7 m by 10 m. **5**
Each of the four sloping faces make an angle of 50° with the horizontal.
The midpoint of *EF* lies vertically above intersection of the diagonals *AC* and *BD*.



Calculate the total surface area of the sloping faces, correct to one decimal place.

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Section II

Part E **15 marks**
Attempt Questions 31–36

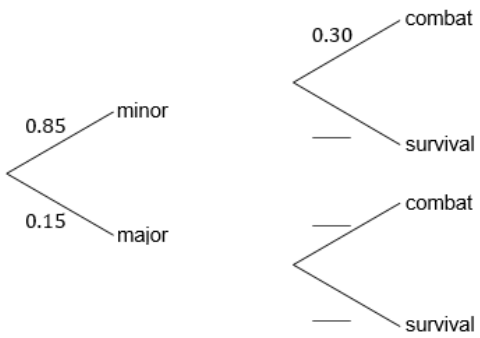
Answer each question in the space provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 31 (3 marks)

In an online game there are various 'power-ups' that players can discover while progressing through the game. The power-ups come in two sizes, major and minor, and they all belong to one of two categories, either combat or survival.
It is known that 85% of the power-ups in the game are minor and the rest are major.
Of the major power-ups it is known that 60 % are combat power-ups, while of the minor power-ups it is known that 30 % are combat power-ups.

(a) Using the given information, complete the tree diagram below. **1**



(b) Power-ups that are discovered in the game occur at random.

(i) Find the probability that when a power-up is discovered it will be a combat power-up. **1**

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(ii) Given that a power-up is a combat power-up, find the probability that it is a major power-up **1**

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Question 32 (3 marks)

Astronomers classify the brightness of stars according to a scale of magnitudes. The difference in magnitude between two stars is defined by the formula

$$m_1 - m_2 = 2.5\log_{10}\left(\frac{b_2}{b_1}\right),$$

where m_1 and m_2 are the magnitudes of the two stars, and b_1 and b_2 are the corresponding apparent brightnesses measured in watts per metre squared.
The magnitude of a star is a unitless measure, and its value can be positive or negative.

The star Sirius has a magnitude of -1.4 and an apparent brightness of 1.04×10^{-7} W/m^2.

(a) The star Acrux has an apparent brightness of 1.42×10^{-8} W/m^2. **1**
Calculate the magnitude of Acrux, correct to two decimal places.

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(b) The Sun has a magnitude of -26.7. **2**
Calculate the apparent brightness of the Sun, correct to three significant figures.

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Question 33 (2 marks)

Doctor Hopper is investigating the population of kangaroos in New South Wales. **2**
The doctor found, based on historical data that the population of kangaroos, P, could be modelled by

$$P = 7500 + 500\left(1.09\right)^t,$$

where t is the number of years since the start of the year 2000, $t \geq 0$.
Find the rate at which the population of kangaroos is growing at the end of 2021.
Leave your answer correct to two significant figures.

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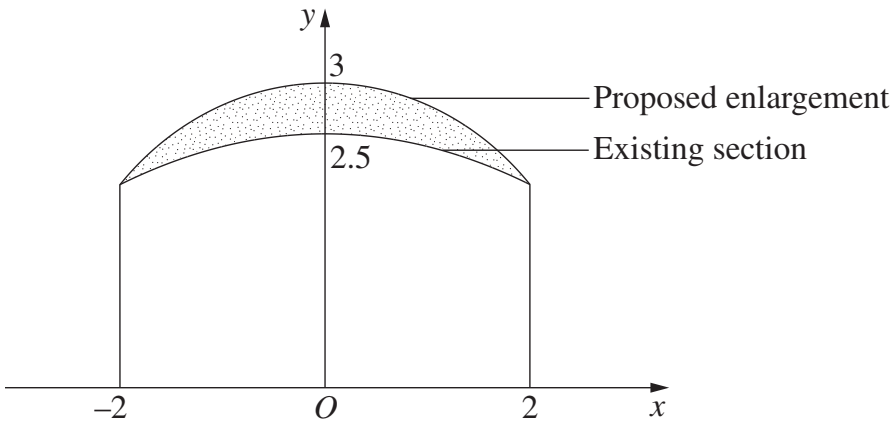
Question 34 (3 marks)

The diagram shows the cross-section of a tunnel and a proposed enlargement. **3**



The heights, in metres, of the existing section at 1 metre intervals are shown in Table *A*.

Table *A*: Existing heights

x	−2	−1	0	1	2
y	2	2.38	2.5	2.38	2

The heights, in metres, of the proposed enlargement are shown in Table *B*.

Table *B*: Proposed heights

x	−2	−1	0	1	2
y	2	2.78	3	2.78	2

Use the trapezoidal rule with the measurements given to calculate the approximate increase in area.

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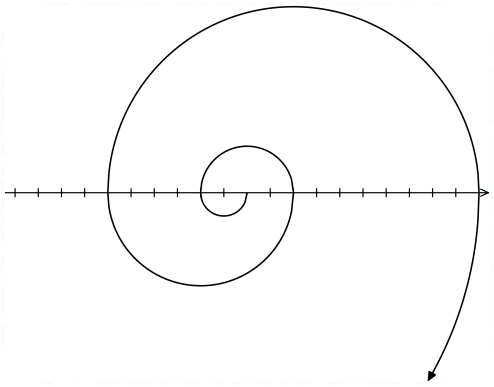
Question 35 (2 marks)

The spiral below is formed by connecting semi-circles of increasing radii. **2**



The lengths of each spiral form a geometric sequence, with the length of the first and second spirals being $\frac{\pi}{2}$ cm and π cm respectively.

Show that the sum of the first n spirals is $\frac{\pi\left(2^n-1\right)}{2}$ cm, where $n \in \mathbb{Z}^+$.

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Question 36 (2 marks)

Evaluate $\int_0^1 \frac{3}{2^{2t}}\,dt$ **2**

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Section II

Part F 15 marks
Attempt Questions 37–39

Answer each question in the space provided. Blank pages are provided at the end of this question to allow rewriting of a part.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 37 (5 marks)

Kay borrows \$60 000 to set up a clothing pop up shop.
She must pay 12% p.a. interest compounded monthly, with repayments at the end of every six months. Let $\$ A_n$ be the amount still owed after the nth repayment of $\$M$.

(a) Show that $A_2 = 60\,000(1.01)^{12} - M(1+1.01^6)$ **2**

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(b) If it takes 15 years to pay off the loan, find the amount of each repayment, M. **3**

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Question 38 (5 marks)

A particle moves along a straight line so that it is x metres to the right of a fixed point O at time t seconds. The acceleration of the particle is given by

$$\ddot{x} = -\frac{2\pi}{3}\sin\frac{\pi}{3}t \text{ m/s}^2$$

Initially the particle is travelling with a velocity of 3 m/s.

(a) Find the first two times when the particle is stationary. **2**

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(b) How far does the particle travel in the first four seconds. **3**

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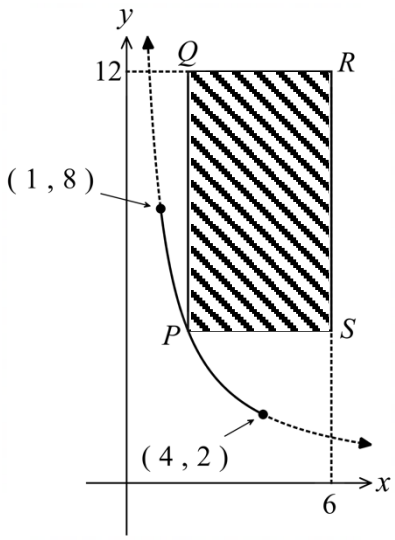
Question 39 (5 marks)

The diagram below shows a rectangle *PQRS*, formed according to the following conditions: **5**

- It is bounded by the lines $x = 6$ and $y = 12$.
- *P* lies on the curve $y = \dfrac{8}{x}$ for $x \in [1, 4]$.
- *R* is the point $(6, 12)$.



Let the area of *PQRS* be *A* square units.

Find the greatest and least possible values of *A* and the corresponding values of *x* for which they occur.

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End of paper

SYDNEY BOYS HIGH SCHOOL

2022

YEAR 12
TASK 4
TRIAL HSC

Mathematics Advanced

Sample Solutions

NOTE: Some of you may be disappointed with your mark.

This process of checking your mark is NOT the opportunity to improve your marks.
Improvement will come through further revision and practice, as well as reading the solutions and comments.

Before putting in an appeal re marking, first consider that the mark is not linked to the amount of writing you have done.

Just because you have shown 'working' does not justify that your solution is worth any marks.

MC Answers

1	A	**6**	D
2	C	**7**	C
3	D	**8**	D
4	B	**9**	A
5	B	**10**	C

Multiple Choice Solutions

1 **A**

$\text{time} \times \text{people} = 35 \times 6 = (7 \times 6) \times 5$

2 **C**

Distance = 48 + (48 – 12) = 60 m

3 **D**

$$\int xe^{x^2}\,dx = \frac{1}{2}\int \underbrace{2x}_{f'(x)}\underbrace{e^{x^2}}_{e^{f(x)}}\,dx = \frac{1}{2}e^{x^2} + C$$

4 **B**

$$-5 \le 5\cos\left(3x + \frac{\pi}{6}\right) \le 5$$

$$\therefore -3 \le 2 + 5\cos\left(3x + \frac{\pi}{6}\right) \le 7$$

5 **B**

Mr Kent has 2 choices out of three of meeting Mr Wayne.
Then Mr Wayne has 1 choice out of 2 to be in the same road as Mr Kent.

$$\frac{2}{3} \times \frac{1}{2} = \frac{1}{3}$$

6 **D**

This is for all values of x.

7 **C**

$$u_{11} = 0.3 \times u_{10} + 6 = 9$$
$$u_{12} = 0.3 \times u_{11} + 6 = 8.7$$

8 D

Shift left first by 2 then down by 1.
It must be done in this order.

9 **A**

NOT TO SCALE

$$\int \frac{2x}{2x-1}\,dx = \int \frac{(2x-1)+1}{2x-1}\,dx$$

$$= \int 1+\frac{1}{2x-1}\,dx$$

$$= \int 1+\frac{1}{2}\times\frac{2}{2x-1}\,dx$$

$$= x+\frac{1}{2}\ln|\,2x-1\,|+C$$

10 C

By considering symmetry, C is the only right answer.

Section II

Part A

Question 11: Solve $2^x = \dfrac{1}{16}$

$2^x = \dfrac{1}{2^4}$

$2^x = 2^{-4} \qquad \therefore x = -4$

Marker's Comment: This question was well done.

Question 12: Factorise $x^3 + 27$

$$\begin{aligned} x^3 + 27 &= (x^3 + 3^3) \\ &= (x+3)(x^3 - 3x + 3^2) \qquad \text{Using: } (a^3 + b^3) = (a+b)(a^2 - ab + b^2) \\ &= (x+3)(x^3 - 3x + 9) \end{aligned}$$

Marker's Comment: This question was poorly done. Some students got the signs wrong

Question 13: Simplify $\dfrac{x}{2} - \dfrac{x-1}{3}$

$$\begin{aligned} &= \frac{3x}{6} - \frac{2(x-1)}{6} \\ &= \frac{3x - 2x + 2}{6} \\ &= \frac{x+2}{6} \end{aligned}$$

Marker's Comment: This question was well done. Marks were deducted for not simplifying fully

Question 14: Find a primitive of $\sqrt{x}$

$y = x^{\frac{1}{2}} \qquad \therefore \int y = x^{\frac{1}{2}}\, dx$

$$\begin{aligned} &= \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \\ &= \frac{2}{3}x^{\frac{3}{2}} + C \\ &= \frac{2}{3}\sqrt{x^3} + C \\ &= \frac{2}{3}x\sqrt{x} + C \end{aligned}$$

Marker's Comment: Well done. Some students failed with algebraic simplification

Question 15: Solve $\cos\theta = -\frac{1}{2}$

θ in Q2, 3

$$\begin{aligned}
\text{Acute angle} &= \cos^{-1}\frac{1}{2} \\
&= \frac{\pi}{3} \\
\text{Q2:}\quad \theta &= \pi - \frac{\pi}{3} \\
&= \frac{2\pi}{3} \\
\text{Q3:}\quad \theta &= \pi + \frac{\pi}{3} \\
&= \frac{4\pi}{3}
\end{aligned}$$

Marker's Comment: Poorly done. Most got the acute angle but failed to get the quadrant 2 & 3 angles. If the domain is given in radians - the answers should be given in radians.

Question 16: Calculate the sum of the arithmetic series $32 + 36 + 40 + \cdots + 144$

$$\begin{aligned}
a = 32, d = 4, T_n &= 144 \\
T_n &= a + (n-1)d \\
144 &= 32 + (n-1)4 \\
144 &= 32 + 4n - 4 \\
4n &= 116 \\
n &= 29 \\
S_n &= \frac{n}{2}[2a + (n-1)d] \\
S_{29} &= 14.5[64 + 28 \times 4] \\
&= 14.5(176) \\
&= 2552
\end{aligned}$$

Marker's Comment: Poorly done. Some students incorrectly calculated n. You could also use the "first plus last" formula for the final calculation..

Question 17 (4 marks)

Differentiate the following with respect to x

(a) $x \tan 2x$ 2

$y = x \cdot \tan 2x$

$y' = 1 \cdot \tan 2x + x \cdot (\sec^2 2x)(2)$ ---(1)

$= \tan 2x + 2x \sec^2 2x$ (1)

(b) $\frac{\ln x}{x^2}$ 2

$y = \frac{\ln x}{x^2}$

$= \ln x (x^{-2})$

$y' = -2x^{-3} \ln x + x^{-2}\left(\frac{1}{x}\right)$

$= -\frac{2 \ln x}{x^3} + \frac{1}{x^3}$

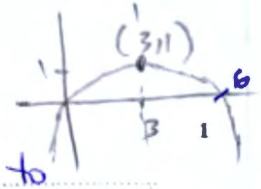
$y' = \frac{1 - 2\ln x}{x^3}$

Quotient Rule works also

Question 18 (3 marks)

The parabola $y = ax^2 + bx + c$ has a vertex at $(3, 1)$ and passes through $(0, 0)$

(a) Find the other x-intercept



Curve must be concave down to go through (0,0). By symmetry about the vertex – the other zero is $x = 6$

(b) Find a, b and c 2

zeros are $x=0$ & $x=6$ so

$y = a(x-0)(x-6)$

$y = ax(x-6)$

(3,1) satisfies equation so

$1 = 3a(-3)$

$1 = -9a$

$a = -\frac{1}{9}$ (1)

so equation is $y = -\frac{x}{9}(x-6)$

$y = -\frac{x^2}{9} + \frac{2}{3}x$

so using $y = ax^2 + bx + c$

$a = -\frac{1}{9}$, $b = \frac{2}{3}$, $c = 0$

Feedback:

Q17a: Reasonably well done. Some students weak on the Product Rule & trig differentiation.

Q17b: Quotient Rule poorly done. Some students failed to fully simplify.

Q18a: Students poor in interpreting graph.

Q18b: Poorly done. Most students got c=0 easily but struggled with a & b values. Many missed the "a" in the * equation in the solution. Many errors were made in number management.

End of Part A

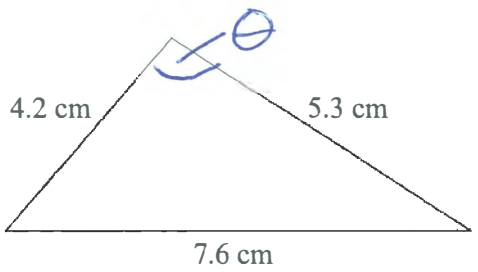
Section II

Part B **15 marks**
Attempt Questions 19–23

Answer each question in the space provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 19 (3 marks)



The lengths of the sides of a triangle are 4.2 cm, 5.3 cm, and 7.6 cm.

(a) Calculate the size of the largest angle of the triangle. **2**
Leave your answer correct to the nearest degree.

$\cos\theta = \dfrac{4.2^2 + 5.3^2 - 7.6^2}{2 \times 4.2 \times 5.3}$

$\theta = 105^\circ$

* Generally well done, no marks deducted for incorrect rounding / not rounding.

(b) Hence, calculate the area of the triangle. **1**
Leave your answer correct to one decimal place.

Area $= \frac{1}{2} \times 4.2 \times 5.3 \times \sin 106$

$A = 10.7 \text{ cm}$

Question 20 (5 marks)

Two events A and B are such that $P(A) = p$, $P(B) = 2p$, $P(A \cup B) = 0.42$, and $P(A \cap B) = 0.03$.

* Generally well done.

(a) Show that $p = 0.15$ **1**

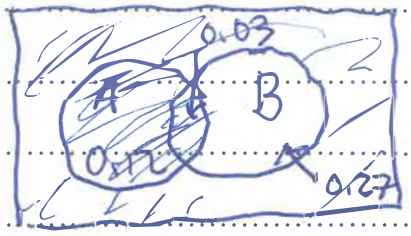
$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$P(A \cup B) = p + 2p - 0.03 = 0.42$

$3p = 0.45 \Longrightarrow p = 0.15$

(b) By use of a Venn diagram, or otherwise, show that $P(A \cup \tilde{B}) = 0.73$, **2**

where $\tilde{B}$ is the complement of B.



$P(\tilde{B}) = 1 - 0.3 = 0.7$

$P(A \cup \tilde{B}) = P(\tilde{B}) + P(A \cap B)$

$P(A \cup \tilde{B}) = 0.7 + 0.03$

Not well done.

(c) Show that $P(A \cap \tilde{B}) = 0.12$ and hence determine, with reasoning, whether the **2**

events A and $\tilde{B}$ are independent.

$P(A \cap \tilde{B}) = P(A) - P(A \cap B) = 0.15 - 0.03 = 0.12$

$P(A \cap \tilde{B}) = 0.12$

$P(A) \times P(\tilde{B}) = 0.15 \times 0.7 = 0.105$

$\therefore P(A \cap \tilde{B}) \neq P(A) \times P(\tilde{B})$

So A and $\tilde{B}$ are NOT independent.

Question 21 (2 marks)

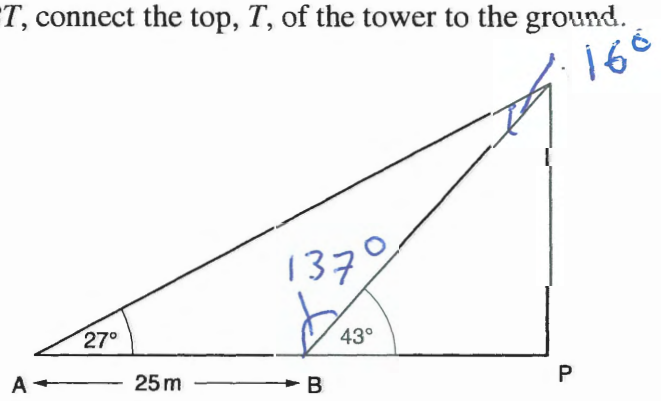
Evaluate $\int_{-1}^{1} x^2 + x \, dx$. **2**

$\left[\frac{x^3}{3} + \frac{x^2}{2}\right]_{-1}^{1} = \frac{1}{3} + \frac{1}{2} - \left(-\frac{1}{3} + \frac{1}{2}\right)$

$= \frac{2}{3}$

Question 22 (2 marks)

TP is a tower on horizontal ground, *ABP*. Note $\angle TPA = 90°$ **2**
Two straight cables, *AT* and *BT*, connect the top, *T*, of the tower to the ground.



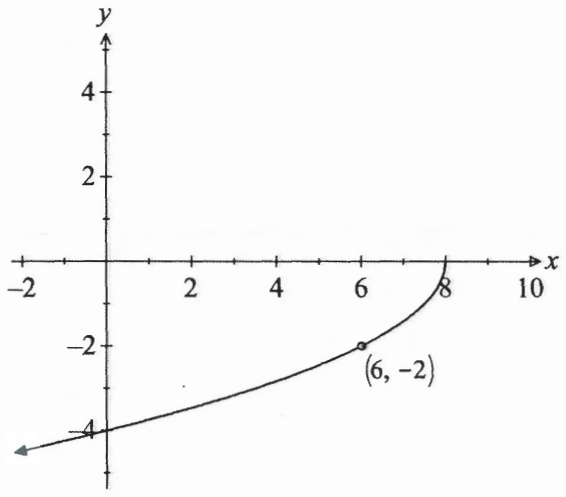
AT and *BT* make angles with the ground of 27° and 43° respectively.
BT is 25 m closer to the foot of the tower, *P*, than *A*.
What is the length of the cable *AT*, correct to 2 significant figures?

$\frac{AT}{\sin 137} = \frac{25}{\sin 16}$

$AT = 62\text{ m}$

Question 23 (3 marks)

The function $f(x) = \sqrt{x}$ is transformed and the equation of the new function is of the form **3**
$y = kf(ax + b)$, where *k*, *a*, and *b* are integer constants. The graph of the new function is shown below.



What are the values of *k*, *a*, and *b*?

$k = -1$, $a = -2$, $b = 16$

$\times -1$ $\div 2$ $+16$

Section II

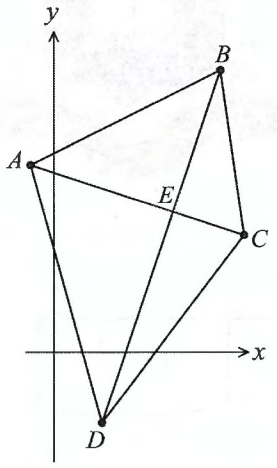
Part C **14 marks**
Attempt Questions 24–27

Answer each question in the space provided.

Your responses should include relevant mathematical reasoning and/or calculations.

Question 24 (7 marks)

A quadrilateral has vertices $A(-1,8)$, $B(7,12)$, $C(8,5)$, and $D(2,-3)$ as shown below.



NOT TO SCALE

Or sub both pts B & D that satisfy the eqn

or (7,12)
$y = mx + c$
$12 = 7 \times 3 + c$
$12 = 21 + c$
$c = -9$ $\therefore y = 3x - 9$

(a) Show that the equation of diagonal BD is $y = 3x - 9$. **2**

B(7,12) D(2,-3) Well Done.

$m_{BD} = \frac{-3-12}{2-7} = \frac{-15}{-5} = 3$ ✓ gradient

$y - 12 = 3(x - 7)$ ✓ working out for the eqn.

$y - 12 = 3x - 21$ $\therefore y = 3x - 9$ RTP.

(b) The equation of diagonal AC is $x + 3y = 23$. Very well Done. **2**
Find the coordinates of E, the point of intersection of the diagonals.

$x + 3(3x - 9) = 23$

$x + 9x - 27 = 23$ ✓ working out

$10x = 50$

$\therefore x = 5$ and $y = 3 \times 5 - 9$

$y = 6$

$\therefore E = (5,6)$ ✓ (½ for each coordinate)

Question 20 continues on page 18

4

Students need to revise their work on coordinate geometry. This is question is not done well.

Question 24 (continued)

(c) Show that E lies on the perpendicular bisector of AB. **3**

$m_{AB} = \frac{12-8}{7--1} = 4/8 = 1/2$ $\frac{1}{2}$

$\therefore$ the gradient of the bisector is -2 $\frac{1}{2}$

midpt AB $= \left(\frac{-1+7}{2}, \frac{8+12}{2}\right) = (3, 10)$ $\frac{1}{2}$

$\perp$ bisector $y-10 = -2(x-3)$

$y-10 = -2x+6$

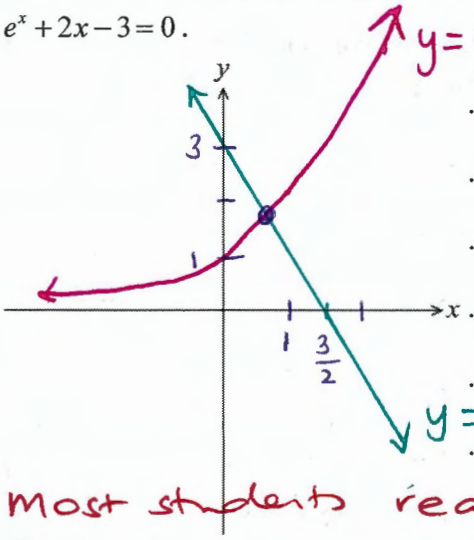
$\therefore 2x+y-16=0$ ✓ with working

$E(5,6) \rightarrow 2(5)+6-16=0$ $\frac{1}{2}$

$\therefore$ E lies on the $\perp$ bisector of AB

Question 25 (2 marks)

By drawing two appropriate graphs determine how many roots there are of the equation $e^x + 2x - 3 = 0$. **2**



$e^x = 3-2x$

↳ $x=0, y=3$

$y=0, 2x=3$

$x=3/2$

$\therefore$ 1 root ($\because$ one POI)

Just state 1 root ✓ $\frac{1}{2}$

① for 2 graphs

$y=e^x$, $y=3-2x$

or $y=e^x-3$, $y=2x$

or $y=-e^x$, $y=2x-3$

* Most students realised the e^x and $2x-3$ split BUT failed to realise $y=-2x+3$.

Question 26 (2 marks)

By using the Reference sheet, or otherwise, find $\int \frac{x^2}{\left(8-x^3\right)^{\frac{1}{2}}}\,dx$. **2**

$\int x^2(8-x^3)^{-1/2}$

$f(x)$

① $\ln \int f'(x)[f(x)]^n dx$ form without changing the question

$= -\frac{1}{3}\int -3x^2(8-x^3)^{-1/2}\,dx$

$= -\frac{1}{3}\,\frac{(8-x^3)^{1/2}}{1/2} + C$ ✓

OK no +c and not in radical form

$= -\frac{2}{3}\sqrt{8-x^3} + C$ ✓ ①

Poorly done. Students need to understand how to apply the reverse chain rule

Question 27 (3 marks)

The table below shows the future value of a $1 annuity.

Future value of $1						
End of Year	1%	2%	3%	4%	5%	6%
1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
2	2.0100	2.0200	2.0300	2.0400	2.0500	2.0600
3	3.0301	3.0604	3.0909	3.1216	3.1525	3.1836
4	4.0604	4.1216	4.1836	4.2465	4.3101	4.3746

(a) What would be the future value of a $24 000 per year annuity at 5% per annum, for two years, with interest compounding annually? **1**

$24000 × 2.0500

= $49200 ✓

(b) An annuity of $31 800 is invested every half-year at 4% per annum, compounded six-monthly for two years. What is the future value of the annuity? **1**

→ 2%

↳ n = 4

$31800 × 4.1216

= $131066.88 ✓

½ for 31800 × 2.0400

= $64872 → ½

(c) Victoria aims to have a deposit for an apartment of at least $92 000 in four years time by investing in an annuity. The annuity has an interest rate of 6% per annum compounded annually. **1**

Calculate Victoria's yearly contribution, to the nearest dollar, to achieve the deposit.

$P \times 4.3746 > 92000$

$P > \$21030.49$ ∴ round up ∵ at least

∴ Yearly contribution = $21031 ✓

½ for 92000×4.3746

3

Must students only received ½ mark ∵ $21030 will not yield $92000

Question 28

$$A = \int_{-2}^{4} \left((-2x^2 + 6x + 11) - (2x - 5) \right) dx$$

$$= \int_{-2}^{4} (-2x^2 + 4x + 16)\, dx$$

$$= \left[-\frac{2}{3}x^3 + 2x^2 + 16x \right]_{-2}^{4}$$

$$= \left(-\frac{128}{3} + 32 + 64\right) - \left(\frac{16}{3} + 8 - 32\right)$$

$$= 120 - \frac{144}{3}$$

$$= 72 \text{ u}^2$$

Notes:

1) Remember to write dx after the integrand.
2) Simplify the integrand before integrating.
3) No need for absolute value; the line is below the parabola between the limits.

Question 29

(a) Substitute $y = 2x + 5$ into the circle equation.

$$x^2 + (2x+5)^2 - 6x - 2(2x+5) - 30 = 0$$

$$x^2 + 4x^2 + 20x + 25 - 6x - 4x - 10 - 30 = 0$$

$$5x^2 + 10x - 15 = 0$$

$$5(x^2 + 2x - 3) = 0$$

$$(x+3)(x-1) = 0$$

$$x = -3, 1.$$

So at Q, $x = 1$, $y = 2(1) + 5 = 7$

$\therefore Q = (1, 7)$.

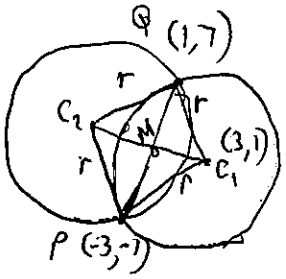
Note: Use substitution to eliminate variable y.

Question 29

(b) From (a), $(x^2-6x+9)+(y^2-2y+1) = 9+1+30$ completing the square

$$(x-3)^2+(y-1)^2 = 40$$

So the centre is $(3, 1)$.



The distance from the centres, C_1, C_2, to P or Q is $r = \sqrt{40}$.

$\therefore PC_1QC_2$ is a rhombus.

So the diagonals bisect each other at M

For PQ, $M = \left(\frac{1-3}{2}, \frac{7-1}{2}\right) = (-1, 3)$.

&For C_1C_2, $(-1,3) = \left(\frac{x+3}{2}, \frac{y+1}{2}\right)$ where $C_2 = (x,y)$ and $C_1 = (3,1)$.

$\therefore x = 2(-1)-3 = -5.$

$y = 2(3)-1 = 5.$

$\therefore$ The equation of the circle is $(x+5)^2+(y-5)^2 = 40.$

Notes

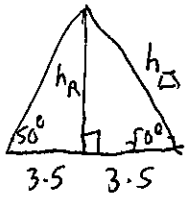
1) Use the given advice about using a rhombus if possible.

Question 30

We need the height of the trapezium face and of the triangle face, plus the length EF.

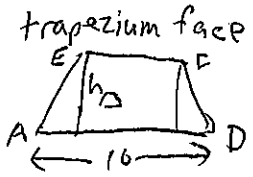
Find height of trapezium face:

Vertical cross-section, looking from side:



$$\cos 50^\circ = \frac{3.5}{h_\Delta}$$

$$h_\Delta = \frac{3.5}{\cos 50^\circ}$$



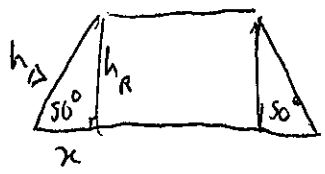
Vertical height of roof:

$$\tan 50^\circ = \frac{h_R}{3.5}$$

$$h_R = 3.5 \tan 50^\circ$$

Find height of triangle face:

Vertical cross-section, from front.



$$\sin 50^\circ = \frac{h_R}{h_\Delta}$$

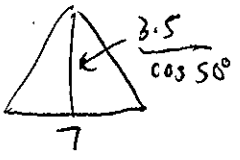
$$h_\Delta = \frac{h_R}{\sin 50^\circ} = \frac{3.5 \tan 50^\circ}{\sin 50^\circ} = \frac{3.5}{\cos 50} \quad \left(\text{since } \tan 50^\circ = \frac{\sin 50^\circ}{\cos 50^\circ}\right)$$

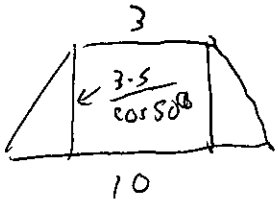
Then x: $\tan 50^\circ = \frac{h_R}{x} = \frac{3.5 \tan 50^\circ}{x}$

So $x = 3.5$.

Then $EF = 10 - 2 \times 3.5 = 3$,

giving faces:





$$\text{So } A = \frac{3.5}{\cos 50^\circ}\left(\frac{1}{2} \times 2 \times 7 + \frac{1}{2}(3+10) \times 2\right)$$

$$= 108.9 \text{ m}^2 \text{ to 1 d.p.}$$

Notes:

1) The question excluded the base rectangle.
2) Keep your numbers exact until the end.
3) Write degrees where needed.

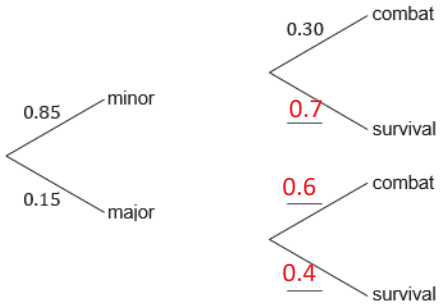
Part E **15 marks**

Your responses should include relevant mathematical reasoning and/or calculations.

Question 31 (3 marks)

In an online game there are various 'power-ups' that players can discover while progressing through the game. The power-ups come in two sizes, major and minor, and they all belong to one of two categories, either combat or survival.
It is known that 85% of the power-ups in the game are minor and the rest are major.
Of the major power-ups it is known that 60 % are combat power-ups, while of the minor power-ups it is known that 30 % are combat power-ups.

(a) Using the given information, complete the tree diagram below. **1**



(b) Power-ups that are discovered in the game occur at random.

(i) Find the probability that when a power-up is discovered it will be a combat power-up. **1**

$0.85\times0.3 + 0.15\times0.6 = 0.345$

(ii) Given that a power-up is a combat power-up, find the probability that it is a major power-up **1**

Let A be major, B be combat

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$= \frac{0.09}{0.345}$$

$$= \frac{6}{23}$$

$$\approx 0.2609$$

[This question was generally very well answered, although many seemed confused by part (b) (ii)]

Question 32 (3 marks)

Astronomers classify the brightness of stars according to a scale of magnitudes. The difference in magnitude between two stars is defined by the formula

$$m_1 - m_2 = 2.5\log_{10}\left(\frac{b_2}{b_1}\right),$$

where m_1 and m_2 are the magnitudes of the two stars, and b_1 and b_2 are the corresponding apparent brightnesses measured in watts per metre squared.
The magnitude of a star is a unitless measure, and its value can be positive or negative.

The star Sirius has a magnitude of -1.4 and an apparent brightness of 1.04×10^{-7} W/m^2.

(a) The star Acrux has an apparent brightness of 1.42×10^{-8} W/m^2. **1**
Calculate the magnitude of Acrux, correct to two decimal places.

$$m_1 = m_2 + 2.5 \log_{10} \frac{b_2}{b_1}$$

$$= -1.4 + 2.5 \log_{10}\left(\frac{1.04 \times 10^{-7}}{1.42 \times 10^{-8}}\right)$$

$$= 0.76$$

[The calculator work was beyond about 50% of candidates.]

(b) The Sun has a magnitude of -26.7. **2**
Calculate the apparent brightness of the Sun, correct to three significant figures.

Making b_1 the subject, let: $A = \dfrac{m_2 - m_1}{2.5}$

$$b_1 = b_2 \times 10^A$$

$$= 1.04 \times 10^{-7} \times 10^{10.12}$$

$$\approx 1370$$

[Fewer than 20% of candidates were able to work this out.]

Question 33 (2 marks)

Doctor Hopper is investigating the population of kangaroos in New South Wales. **2**
The doctor found, based on historical data that the population of kangaroos, P, could be modelled by

$$P = 7500 + 500\left(1.09\right)^{t},$$

where t is the number of years since the start of the year 2000, $t \geq 0$.
Find the rate at which the population of kangaroos is growing at the end of 2021.
Leave your answer correct to two significant figures.

$$\frac{dP}{dt} = 500 \times (1.09)^{t} \ln 1.09$$

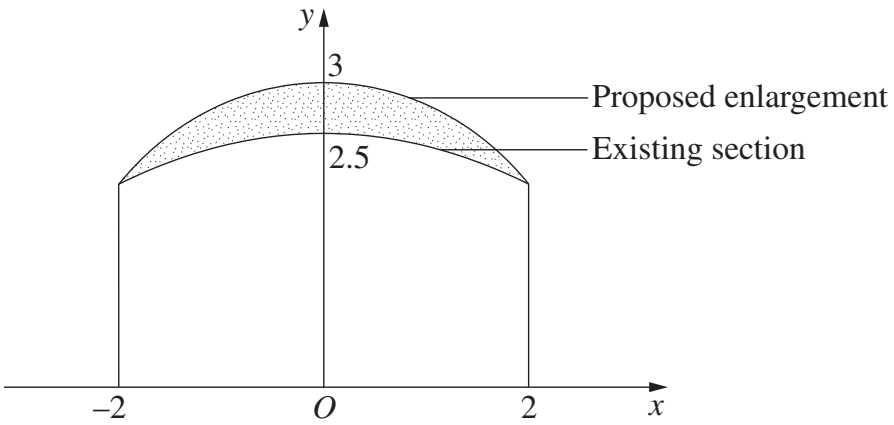
When $t = 22$

$$\frac{dP}{dt} = 500 \times (1.09)^{22} \ln 1.09$$
$$\simeq 290 \text{ per year}$$

[Most candidates got the number of years wrong, and were penalised.
About 25% failed to take the derivative.]

Question 34 (3 marks)

The diagram shows the cross-section of a tunnel and a proposed enlargement. **3**



The heights, in metres, of the existing section at 1 metre intervals are shown in Table *A*.

Table *A*: Existing heights

x	−2	−1	0	1	2
y	2	2.38	2.5	2.38	2

The heights, in metres, of the proposed enlargement are shown in Table *B*.

Table *B*: Proposed heights

x	−2	−1	0	1	2
y	2	2.78	3	2.78	2

Use the trapezoidal rule with the measurements given to calculate the approximate increase in area.

Current Area = ½ (2 + 2(2.38 + 2.5 + 2.38) + 2)

= 9.26

Proposed Area = ½ (2 + 2(2.78 + 3 + 2.78) + 2)

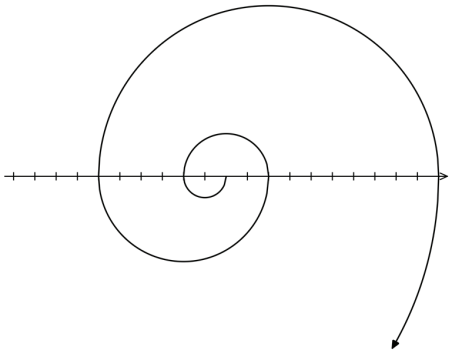
= 10.56

∴ Increase in area = 1.3 sq m

[This was generally well answered, but a small proportion had the trapezoidal formula wrong, not managing to find $h/2$ correctly.]

Question 35 (2 marks)

The spiral below is formed by connecting semi-circles of increasing radii. **2**



The lengths of each spiral form a geometric sequence, with the length of the first and second spirals being $\frac{\pi}{2}$ cm and π cm respectively.

Show that the sum of the first n spirals is $\frac{\pi\left(2^n-1\right)}{2}$ cm, where $n \in \mathbb{Z}^+$.

$a = \frac{\pi}{2}, r = 2$

$S_n = \frac{a\left(r^n-1\right)}{r-1}$

$= \frac{\pi}{2} \times \frac{2^n-1}{2-1}$

$= \frac{\pi\left(2^n-1\right)}{2}$

[Very well answered.]

Question 36 (2 marks)

Evaluate $\int_0^1 \frac{3}{2^{2t}}\,dt$.

$$\begin{aligned}\int_0^1 \frac{3}{2^{2t}}\,dt &= 3\int_0^1 2^{-2t}dt \\ &= 3\int_0^1 4^{-t}dt \\ &= -3\int_0^1 -1.4^{-t}dt \\ &= -3\left[\frac{4^{-t}}{\ln 4}\right]_0^1 \\ &= -3\left[\frac{1}{4\ln 4}-\frac{1}{\ln 4}\right] \\ &= \frac{9}{4\ln 4}\end{aligned}$$

[About 50% seemed to have no idea, and did not use the reference sheet.

Question 37 (5 marks)

Kay borrows \$60 000 to set up a clothing pop up shop.
She must pay 12% p.a. interest compounded monthly, with repayments at the end of every six months. Let \$ A_n be the amount still owed after the nth repayment of \$$M$.

(a) Show that $A_2 = 60\,000(1.01)^{12} - M(1+1.01^6)$ **2**

12% p.a. = 1% per month
After the 1st month Kay owes $60\,000 \times 1.01$
This goes on, so that at the end of the 5th month Kay owes $60\,000 \times 1.01^5$.
$\therefore A_1 = 60\,000 \times 1.01^6 - M$

At the end of the 7th month Kay owes $(60\,000 \times 1.01^6 - M)1.06$
This goes on, so that at the end of the 11th month Kay owes. $(60\,000 \times 1.01^6 - M)1.06^{11}$

$$\begin{aligned}\therefore A_2 &= A_1 \times 1.06^6 - M \\ &= (60\,000 \times 1.01^6 - M) \times 1.01^6 - M \\ &= 60\,000 \times 1.01^{12} - M(1+1.01^6)\end{aligned}$$

Comment: Many students 'reverse engineered' A_2 i.e. the A_2 was not shown.

Question 37 (continued)

(b) If it takes 15 years to pay off the loan, find the amount of each repayment, M. **3**

15 years = 30 repayments

$$\begin{aligned} A_3 &= A_2 \times 1.01^6 - M \\ &= \left[60\,000 \times 1.01^{12} - M\left(1+1.01^6\right) \right] \times 1.01^6 - M \\ &= 60\,000 \times 1.01^{12} - M\left(1+1.01^6+1.01^{12}\right) \end{aligned}$$

$$\therefore A_{30} = 60\,000 \times 1.01^{6\times 30} - M\left(1+1.01^{6\times 1}+1.01^{6\times 2}+\cdots+1.01^{6\times 29}\right) \quad \textbf{(*)}$$

$$= 60\,000 \times 1.01^{180} - M\left(\underbrace{1+1.01^6+1.01^{12}+\cdots+1.01^{174}}_{a=1,\, r=1.01^6,\, n=30} \right)$$

$$= 60\,000 \times 1.01^{180} - M \times \frac{\left(1.01^6\right)^{30}-1}{1.01^6-1} \quad \textbf{(**)}$$

$$A_{30} = 0 \Rightarrow 0 = 60\,000 \times 1.01^{180} - M \times \frac{\left(1.01^6\right)^{30}-1}{1.01^6-1}$$

$$\therefore M = \frac{60\,000 \times 1.01^{180} \times \left(1.01^6-1\right)}{1.01^{180}-1} \approx 4430.07$$

Comment: Common errors that were penalised.

- A_3 has to be shown and not just stated.
- Students need to 'generalise' to the line indicated by (*) and NOT (**)
- Students used $r-1=0.01$ rather than $r-1=(1.01)^6-1$
  In fact, some students started writing $r=1.06$ or $(1.06)^6$.
- Calculator mistakes.
- Many students calculated A_{15} rather than A_{30}.
  This was a significant mistake as the student hadn't realised what part (a) was about or they just didn't get it.
  A student who made no other mistakes could only get a maximum of 2 marks if they calculated A_{15}.

Question 38 (5 marks)

A particle moves along a straight line so that it is x metres to the right of a fixed point O at time t seconds. The acceleration of the particle is given by

$$\ddot{x} = -\frac{2\pi}{3}\sin\frac{\pi}{3}t \text{ m/s}^2$$

Initially the particle is travelling with a velocity of 3 m/s.

(a) Find the first two times when the particle is stationary. **2**

$$\ddot{x} = -\frac{2\pi}{3}\sin\frac{\pi}{3}t$$

$$\dot{x} = -\frac{2\pi}{3}\times -\frac{3}{\pi}\cos\frac{\pi}{3}t + C$$

$$= 2\cos\frac{\pi}{3}t + C$$

$t = 0, v = 3$: $\quad 3 = 2\cos 0 + C$

$$\therefore C = 1$$

$$\dot{x} = 1 + 2\cos\frac{\pi}{3}t$$

The particle is stationary when $v = 0$

$$\therefore 2\cos\frac{\pi}{3}t + 1 = 0$$

$$\therefore \frac{\pi}{3}t = \frac{2\pi}{3}, \frac{4\pi}{3}$$

$$\therefore t = 2, 4$$

Comment: Some students interpreted $\sin\frac{\pi}{3}t$ as $\left(\sin\frac{\pi}{3}\right)t$.

They should have realised their mistake when the question asked "Find the first two times ..."

A lot of students did not calculate C.

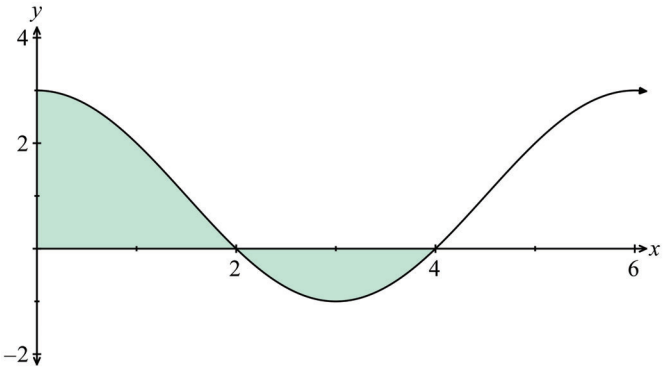
A lot of students need to revise how to solve trigonometric equations

Setting out was appalling. Students are given a space to re-write their answers.

Question 38 (continued)

(b) How far does the particle travel in the first four seconds. **3**

The particle changes velocity from positive to negative at $t = 2$



$$\text{Distance} = \int_0^2 1+2\cos\frac{\pi}{3}\,dt + \left|\int_2^4 1+2\cos\frac{\pi}{3}\,dt\right|$$

$$= \left[t+\frac{6}{\pi}\sin\frac{\pi}{3}t\right]_0^2 + \left|\left[t+\frac{6}{\pi}\sin\frac{\pi}{3}t\right]_2^4\right|$$

$$= 2+\frac{6}{\pi}\times\frac{\sqrt{3}}{2}-0+\left|4-\frac{6}{\pi}\times\frac{\sqrt{3}}{2}-\left(2+\frac{6}{\pi}\times\frac{\sqrt{3}}{2}\right)\right|$$

$$= 2+\frac{3\sqrt{3}}{\pi}+\left|2-\frac{6\sqrt{3}}{\pi}\right| = \frac{9\sqrt{3}}{\pi} \approx 4.96$$

Comment: The question asks for the distance travelled and not the change in displacement.

Students who calculated, with no other mistakes $\int_0^4\left(2\cos\frac{\pi}{3}t+1\right)dt$ could only get a maximum of 2 marks.

These students should have wondered what implications part (a) brought to the overall question.

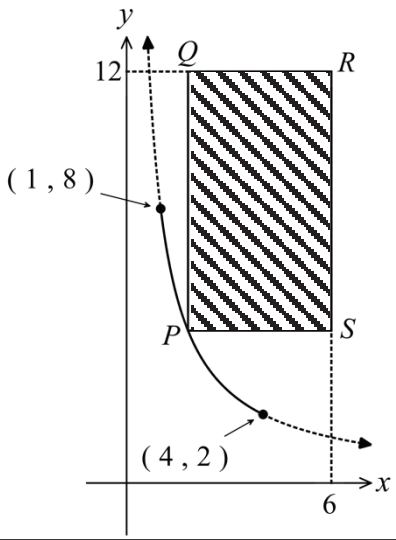
It was very hard for students to gain marks if they had interpreted $\sin\frac{\pi}{3}t$ as $\left(\sin\frac{\pi}{3}\right)t$.

Setting out was appalling. Students are given a space to re-write their answers.

Question 39 (5 marks)

The diagram below shows a rectangle *PQRS*, formed according to the following conditions. **5**

- It is bounded by the lines $x = 6$ and $y = 12$.
- *P* lies on the curve $y = \dfrac{8}{x}$ for $x \in [1, 4]$.
- *R* is the point $(6, 12)$.



Let the area of *PQRS* be *A* square units.

Find the greatest and least possible values of *A* and the corresponding values of *x* for which they occur.

Let the point *P* be $P(x, y)$

$$\begin{aligned} A &= QR \times QP \\ &= (6 - x)(12 - y) \\ &= 72 - 6y - 12x + xy \\ &= 72 - \frac{48}{x} - 12x + 8 \\ &= 80 - 12x - \frac{48}{x} \end{aligned}$$

$$\frac{dA}{dx} = -12 + \frac{48}{x^2}$$

$$\frac{d^2A}{dx^2} = -\frac{96}{x^3} \qquad \left(< 0 \text{ for } x \in [1, 4]\right)$$

Stationary points when $\dfrac{dA}{dx} = 0$

$$\therefore -12 + \frac{48}{x^2} = 0$$

$$\therefore x^2 = 4$$

$$\therefore x = 2 \qquad (x \in [1, 4])$$

From above, $x = 2$ is a relative maximum value for $A = 32$

Check the boundary points: $x = 1$, $A = 20$
$x = 4$, $A = 20$

So the least value of *A* is 20 and the greatest is 32.

Comment: This question has nothing to do with integration.

An area answer is required, not just the *x*-coordinates.

Students are reminded that they need to put in (correct) numerical values (or equivalent) with their reasoning. Not just "+" and "–".

Students had to check both $x = 1$ and $x = 4$, or suitable reasons why not. Most students who didn't check just assumed that as $1 < 4$ then $A_1 < A_4$.